Variational symmetries and Noether's theorem on time scales

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The classical theorem of Emmy Noether [E. Noether, 1918] on calculus of variations says that for any variational symmetry of a minimization problem there exists a conserved quantity along the respective Euler-Lagrange extremals. We extend this theorem to the calculus of variations on arbitrary time scales. The differential calculus on time scales unifies differential and difference calculus via the concept of delta derivative, which behaves like the standard derivative $f'(t)$ for the continuous time and like the difference $f(t+1) - f(t)$ for the discrete time. The concept of integral on time scales extends the standard integral and the finite sum. We construct the quantity that is preserved by the variational symmetry. It contains an extra term not present in the classical statement in the continuous time. This term depends on the graininess of the time scale and vanishes when the graininess is 0 (continuous time). The proof of the result is done in two steps. First we prove the theorem for the case when the symmetry transformations do not change time (without transforming the independent variable). Then, using time-reparameterization, we obtain the Noether's theorem on time scales in its general form.