

# MODULI AND NILPOTENT APPROXIMATIONS IN THE MONSTER TOWER

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It is known that Goursat flags (sometimes called also Cartan–Goursat) possess locally universal models. It is so in any length  $r \geq 2$  of such flags: there exists a monster manifold  $\mathcal{M}_r$  of dimension  $r + 2$  with a rank-two Goursat distribution  $\mathcal{D}_r$  living on it, locally modelling any other corank- $r$  Goursat distribution. In fact, the kinematic systems ‘car + trailers’ give rise to very clear examples of monster manifolds. Quite simply, their configuration spaces  $\mathbb{R}^2 \times (S^1)^r$  (when the # of trailers is  $r - 1$ ) are such, together with a properly defined  $\mathcal{D}_r$  describing the moves of the systems. When the parameter  $r$  grows from 2 to infinity, one gets a sequence of monster manifolds – the Monster Tower in the terminology of Montgomery and Zhitomirskii.

By now it is also known that the manifolds  $\mathcal{M}_r$  are stratified into *geometric classes* labelled by words of length  $r$  over the alphabet  $\{G, S, T\}$ , cf. for inst. [1]. Within each fixed stratum  $\mathcal{C} \subset \mathcal{M}_r$ , the small growth vector of  $\mathcal{D}_r$  is constant, not depending on a point. This is an important and elegant result of F. Jean (1996). As a consequence, the nilpotent approximations (NAs) of  $\mathcal{D}_r$  at points of  $\mathcal{C}$  have one and the same small growth vector.

Knowing this, A. Agrachev asked in the year 2000 if the moduli [of the local classification] of Goursat distributions were visible on the nilpotent level.

A recent (2006) result of the author says: in certain cases, not. For instance, not for an involved modulus hidden in the class GGSGSGSG. We want firstly to present the status, resisting explanation for a couple of years, of another modulus residing in the class GGGSGSGGG. Then, secondly, to venture to say that perhaps much more is true about the NAs of  $\mathcal{D}_r$  at points of  $\mathcal{M}_r$ .

**Conjecture.** In any length  $r$ , the NA of  $\mathcal{D}_r$  at a point  $p \in \mathcal{M}_r$  depends solely on the geometric class of  $p$ . In particular, all numerical moduli of points in the Monster Tower *conjecturally* disappear on the level of NAs.

## Literatura

- [1] P. Mormul; *Geometric classes of Goursat flags and the arithmetics of their encoding by small growth vectors*, Central European J. Math. **2** (2004), 859 – 883 (electronic).