Age-dependent single-species population dynamics with delayed argument

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A.G. McKendrick was the first who introduced in 1926 the age-structure to the consideration on the dynamic of a population. He described single-species population in which each individual is capable to the reproduction. It is so called classical von Foerster model based on the following system of the equations

$$\begin{cases} \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = -\lambda(x,t)u \\ u(x,0) = v(x) \\ u(0,t) = \int_0^\infty \beta(x,t)u(x,t)dx. \end{cases}$$

The above system can be generalized in many ways. The example can be the model proposed by M. E. Gurtin and R. C. MacCamy in 1974. It was based on the assumption that the progress of the population depends on its number. Therefore the mortality as well as the reproductive abilities depend on this number. These results are the inspiration for our presentation. It is common knowledge that other factors can have an influence on the progress of the population. Natural generalization of the population dynamics is taking into consideration, for example: two genders, limitation of sources of the natural environment, period of gestation or period of a response of a system to a stimulus. Two last examples suggest the necessity to consider the descriptions with delayed parameter. We will present the generalized Gurtin-McCamy model with the assumption that both, the reproduction as the death, depend on the population in any preceding period of time.

$$\begin{cases} \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = -\lambda(x,z_t)u(x,t) \\ u(x,0) = v(x) \\ u(0,t) = \int_0^\infty \beta(x,z_t)u(x,t)dx \\ z(t) = \int_0^\infty u(x,t)dx \\ z_t = z(t+s). \end{cases}$$

We will present the details of this construction.